Word Processing in Groups

Eric Bailey

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Part 1

An Introduction to Automatic Groups

CHAPTER 1

Finite State Automata, Regular Languages and Predicate Calculus

1. Languages and Regular Languages

- An *alphabet* A is nothing more than a finite set.
  - If A is the *alphabet* over lowercase letters, "automaton" is a *string* over A with $n = 9$.
  - If ω is a *string* $\{1, ..., n\} \to A$, we call n the *length* of ω and we denote it by $|\omega|$.
- An element of a A is called a *letter*.
- A *string* over the *alphabet* A is a finite sequence of *letters*, i.e. an integer $n \geq 0$ and a mapping $\{1, ..., n\} \to A$.
- If $n = 0$, the domain is the nullset and there is a unique string, the *nullstring*, generally denoted ϵ, or sometimes ϵ_A to distinguish the *nullstring* over A, since ϵ might be a *letter* in A, e.g. Definition 1.1.3.
- The set of all *strings* over the *alphabet* A is denoted A^*.
  - With the operation of *concatenation*, the set A^* of *strings* over A forms a *monoid*, with *identity element* ϵ.
  - A^* is the free monoid or *semigroup* on the set of generators A.
- All *semigroups* considered in this book will be *monoids*, so the words are used interchangeably.
- Given two *strings* $\omega : \{1, ..., n\} \to A$ and $\tau : \{1, ..., m\} \to A\}$, the *concatenation* $\omega\tau$ of ω and τ is defined to be the *string* $\{1, ..., m + n\} \to A$ given by $(\omega\tau)(i) = \omega(i)$ if $1 \leq i \leq n$ and $(\omega\tau)(i) = \tau(i - n)$ if $n + 1 \leq i \leq n + m$.

Glossary

alphabet: nothing more than a finite set. 7, 9

binary operation: TODO: write description. 9

concatenation: TODO: write description. 7

identity element: a special type of element of a set, with respect to a *binary operation* on that set, which leaves other elements unchanged when combined. 7

length: TODO: write description. 7

letter: an element of an *alphabet*. 7, 9

monoid: a set with an associative multiplication and an identiy. 7

nullstring: a unique *string* over an *alphabet* A where $n = 0$ and the domain is the nullset, generally denoted ϵ. 7, 9

semigroup: a set with an associative multiplication. 7

string: a finite sequence of *letters*, i.e. an integer $n \geq 0$ and a mapping $\{1, ..., n\} \to A$. 7, 9